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Model-independent determination of the top Yukawa coupling from LHC and ILC

K. Desch[1,a], M. Schumacher[2,b]

[1] Physikalisches Institut, Universität Freiburg, Hermann-Herder-Str. 3, 79104 Freiburg, Germany
[2] Physikalisches Institut der Universität Bonn, Nussallee 12, 53112 Bonn, Germany

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Abstract. We show how a measurement of the process $\mathrm{pp} \to \mathrm{t\bar{t}H^0} + X$ at the Large Hadron Collider (LHC) and a measurement of the Higgs boson branching ratios $\mathrm{BR(H^0 \to b\bar{b})}$ and $\mathrm{BR(H^0 \to W^+W^-)}$ at a future linear electron positron collider (ILC) can be combined to extract a value of the top quark Yukawa coupling in a model-independent way. We find that for masses with $120\,\mathrm{GeV}/c^2 < \mathrm{m_H} < 200\,\mathrm{GeV}/c^2$ a measurement precision of 15% including systematic uncertainties can be achieved for integrated luminosities of $300\,\mathrm{fb}^{-1}$ at the LHC and $500\,\mathrm{fb}^{-1}$ at the ILC at a centre-of-mass energy of 350 GeV.

1 Motivation

The Yukawa coupling g_t of the Higgs boson to the heaviest quark, the top quark, is of great interest for the study of the nature of electroweak symmetry breaking and the generation of masses. While the Yukawa couplings to bottom and charm quarks and to tau leptons and muons are in principle accessible through the Higgs boson decay branching ratios, the Higgs boson decay into top quark pairs is kinematically forbidden for light Higgs bosons as they are favoured by theory and electroweak precision data. The only standard model process that probes the top Yukawa coupling at tree level is the associated production of a $\mathrm{t\bar{t}}$ pair with a Higgs boson. This process occurs at the Large Hadron Collider (LHC) ($\mathrm{pp} \to \mathrm{t\bar{t}H^0}$) [1] as well as at the International Linear Collider (ILC) ($\mathrm{e^+e^-} \to \mathrm{t\bar{t}H^0}$) [2, 3]. In the latter case the cross section is only significant at centre-of-mass energies in excess of 800 GeV. At the LHC, the final states that have been investigated so far are $\mathrm{t\bar{t}b\bar{b}}$ [4–7] and $\mathrm{t\bar{t}W^+W^-}$ [8, 9]. At tree level, their production rates are proportional to $g_\mathrm{t}^2\ \mathrm{BR(H^0 \to b\bar{b})}$ and $g_\mathrm{t}^2\ \mathrm{BR(H^0 \to W^+W^-)}$, respectively. The absolute values of $\mathrm{BR(H^0 \to b\bar{b})}$ and $\mathrm{BR(H^0 \to W^+W^-)}$ can be measured accurately in a model-independent way at the ILC from the corresponding decay branching ratios [10]. These can be measured already at a first phase of the ILC ($\sqrt{s} = 350\,\mathrm{GeV}$). Thus, the combination of the measurements of both machines can be used to determine the value of g_t without model assumptions before the accuracy can be improved in a second phase of the ILC ($\sqrt{s} \sim 1$ TeV) at a later time.

2 Measurements at the LHC

The results from the following ATLAS MC studies of the $\mathrm{t\bar{t}H^0}$ process are used:

1. $\mathrm{t\bar{t}H^0}$ with $\mathrm{t\bar{t} \to b\bar{b}q}\,\ell\nu$ and $\mathrm{H^0 \to b\bar{b}}$ [4];
2. $\mathrm{t\bar{t}H^0}$ with $\mathrm{H^0 \to W^+W^-}$ and two like-sign leptons [8];
3. $\mathrm{t\bar{t}H^0}$ with $\mathrm{H^0 \to W^+W^-}$ and three leptons [8].

The expected numbers of selected signal and background events in the three channels for various Higgs masses and total integrated luminosities of $30\,\mathrm{fb}^{-1}$ collected at low-luminosity running of the LHC ($L = 10^{33}\,\mathrm{cm^{-2}s^{-1}}$) and $300\,\mathrm{fb}^{-1}$ collected at high-luminosity running of the LHC ($L = 10^{34}\,\mathrm{cm^{-2}s^{-1}}$) are listed in Tables 1–3. The results obtained in this section are based on the anticipated data sample of *one* LHC detector, with the luminosity per detector quoted above. Note that the signal and background rates and the signal-to-background ratio differ for the $30\,\mathrm{fb}^{-1}$ and $300\,\mathrm{fb}^{-1}$ samples due to the different anticipated detector performance at low- and high-luminosity running, e.g. the b-tagging performance.

From the expected event numbers we first estimate the uncertainty (statistical and systematic) of the measured cross-section $\sigma^\mathrm{data}_\mathrm{t\bar{t}H}$. Further uncertainties arise when $\sigma^\mathrm{data}_\mathrm{t\bar{t}H}$ is compared to the theoretical prediction as a function of g_t.

The uncertainty of the observed cross section $\sigma^\mathrm{data}_\mathrm{t\bar{t}H}$ is calculated as

$$\left(\frac{\Delta\sigma^\mathrm{data}_\mathrm{t\bar{t}H}}{\sigma^\mathrm{data}_\mathrm{t\bar{t}H}}\right)^2 = \frac{S+B}{S^2} + \left(\frac{\Delta B_\mathrm{syst}}{S}\right)^2 + \left(\frac{\Delta\mathcal{L}}{\mathcal{L}}\right)^2 + \left(\frac{\Delta\epsilon}{\epsilon}\right)^2 .$$

[a] e-mail: desch@physik.uni-freiburg.de
[b] e-mail: schumach@physik.uni-bonn.de

Table 1. Expected number of signal and background events for the $t\bar{t}H^0$ with $t\bar{t}\to bbq\ell\nu$ and $H^0\to b\bar{b}$ analysis at the LHC [4]

m_H	30 fb^{-1}		300 fb^{-1}	
(GeV/c^2)	$t\bar{t}H^0$ $H^0\to b\bar{b}$	background	$t\bar{t}H^0$ $H^0\to b\bar{b}$	background
100	83.4	303.4	279.0	1101.3
110	63.0	275.7	232.5	1140.6
120	43.0	234.1	173.1	1054.2
130	26.5	200.1	112.5	1015.8
140	13.9	178.2	62.4	947.1

Table 2. Expected number of signal and background events for the $t\bar{t}H^0$ with $H^0\to W^+W^-$ (two like-sign leptons and three leptons, respectively) analyses at the LHC [8] for 30 fb^{-1}

	30 fb^{-1}			
m_H	$t\bar{t}H^0$ $H^0\to WW(2\ell)$		$t\bar{t}H^0$ $H^0\to WW(3\ell)$	
(GeV/c^2)	signal	background	signal	background
120	4.4	19.6	2.7	21.2
140	15.0	19.6	8.7	21.2
160	21.1	19.6	13.0	21.2
180	17.3	19.6	10.3	21.2
200	10.5	19.6	5.7	21.2

Table 3. Expected number of signal and background events for the $t\bar{t}H^0$ with $H^0\to W^+W^-$ (two like-sign leptons and three leptons, respectively) analyses at the LHC [8] for 300 fb^{-1}

	300 fb^{-1}			
m_H	$t\bar{t}H^0$ $H^0\to WW(2\ell)$		$t\bar{t}H^0$ $H^0\to WW(3\ell)$	
(GeV/c^2)	signal	background	signal	background
120	12.7	80.6	10.5	97.6
140	50.0	80.6	33.7	97.6
160	72.3	80.6	55.3	97.6
180	60.9	80.6	41.7	97.6
200	43.2	80.6	26.4	97.6

Here, S (B) denotes the number of expected signal (background) events. $\Delta B_{\rm syst}$ is the uncertainty of the background determination from sideband data (10% in the $H^0\to b\bar{b}$ channel at high luminosity, 5% otherwise). $\Delta\mathcal{L}$ is the uncertainty of the integrated luminosity (5%) and $\Delta\epsilon$ is the uncertainty of the determination of the efficiency. This error involves uncertainties of the tagging efficiency for individual b-jets (3%) and leptons (3% from the isolation requirement and 2% from reconstruction efficiency) and an overall detector efficiency uncertainty of 2% (following [11]). The total value of $\Delta\epsilon$ is then calculated for each channel individually depending on the number of leptons and b-jets.

The expected error including systematic uncertainties and taking into account only the statistical error of each channel is shown in Table 4. For the $H^0\to W^+W^-$ decay mode the signal and background from the two-lepton and three-lepton channels are added together since their signal contributions are exclusive and the overlap in the background is small. The obtained result is consistent with the study presented in [11].

In the next step the uncertainty of g_t^2 BR($H^0\to b\bar{b}/W^+W^-$) that arises when the observed $\sigma_{t\bar{t}H}$ BR($H^0\to b\bar{b}/W^+W^-$) is compared to its theoretical prediction. These uncertainties consist of the uncertainties in the proton structure functions (5% [12, 13]) and uncertainties in the calculation of the production cross section. Recent full next-to-leading order (NLO) calculations estimate the uncertainty of the cross-section prediction to be approximately 15% from a variation of the renormalisation and factorisation scales [14–17]. The total theoretical uncertainty $\Delta\sigma_{t\bar{t}H}^{\rm theo}$ is obtained by adding the above two sources in quadrature.

Finally, the total uncertainty Δg_t^2 BR($H^0\to xx$), where xx denotes the Higgs decays $b\bar{b}$ and W^+W^-, is obtained

Table 4. Expected relative precision of $\sigma_{t\bar{t}H}$ BR($H^0 \to X$) for the various LHC $t\bar{t}H^0$ analyses including systematic uncertainties (statistical errors only). For $H^0 \to W^+W^-$ the expected signal and background in the two- and three-lepton final states have been added

m_H	30 fb^{-1}		300 fb^{-1}	
(GeV/c^2)	$H^0 \to b\bar{b}$	$H^0 \to WW$	$H^0 \to b\bar{b}$	$H^0 \to WW$
100	0.398(0.236)		0.249(0.133)	
110	0.476(0.292)		0.287(0.159)	
120	0.598(0.387)	1.023(0.974)	0.345(0.202)	0.732(0.611)
130	0.840(0.568)	0.524(0.492)	0.488(0.299)	0.362(0.295)
140	1.444(0.997)	0.370(0.339)	0.804(0.509)	0.252(0.193)
160		0.287(0.254)		0.196(0.137)
180		0.331(0.300)		0.221(0.163)
200		0.486(0.454)		0.282(0.222)

Table 5. Relative precision of the branching ratio for $H^0 \to b\bar{b}$ and $H^0 \to W^+W^-$ expected for the ILC running at $\sqrt{s} = 350$ GeV with 500 fb^{-1} [10]

m_H (GeV/c^2)	$H^0 \to b\bar{b}$	$H^0 \to W^+W^-$
100	0.024	
120	0.024	0.051
140	0.026	0.025
160	0.065	0.021
200		0.021

according to

$$\left(\frac{\Delta[g_t^2\,\mathrm{BR}(H^0 \to xx)]}{g_t^2\,\mathrm{BR}(H^0 \to xx)}\right)^2 = \left(\frac{\Delta\sigma_{t\bar{t}H}^{\mathrm{theo}}}{\sigma_{t\bar{t}H}^{\mathrm{theo}}}\right)^2 + \left(\frac{\Delta\sigma_{t\bar{t}H}^{\mathrm{data}}}{\sigma_{t\bar{t}H}^{\mathrm{data}}}\right)^2 .$$

3 Measurements at the ILC

At the ILC, the decay branching ratios into b quark pairs and W boson pairs can be measured at $\sqrt{s} = 350$ GeV to the precision listed in Table 5 [10]. The precise model-independent measurement of Higgs boson branching ratios exploits the measurement of the Higgs-strahlung process $e^+e^- \to H^0Z^0$. Events from specific Higgs decays, e.g. $H^0 \to b\bar{b}$ and $H^0 \to W^+W^-$ can be cleanly identified. The branching ratio is determined by normalising the observed rate for a specific Higgs decay to the total Higgs-strahlung rate. The latter can be measured from the selection of $Z^0 \to \ell^+\ell^-$ events where the invariant mass of the recoil system is consistent with the Higgs mass, independent of the Higgs decay.

4 Results

For an extraction of the top quark Yukawa coupling at each Higgs boson mass we combine the LHC rate measurement of $t\bar{t}H^0$ with $H^0 \to b\bar{b}$ or $H^0 \to W^+W^-$ with the corresponding measurement of the branching ratio at the ILC. We make the tree-level assumption that the cross section $\sigma_{t\bar{t}H}$ is proportional to g_t^2. Thus, the relative error in g_{ttH} is simply given by $\Delta g_t/g_t = 0.5\Delta\sigma_{t\bar{t}H}/\sigma_{t\bar{t}H}$. Higher-order electroweak corrections as well as potential interference of the signal process with $b\bar{b}t\bar{t}$ ($b\bar{b}W^+W^-$) processes without Higgs exchange may alter this relation slightly. They can be easily accommodated in a fit of g_t to the measured cross sections. They are not expected to change the estimated uncertainties significantly.

The relative error in σ_{ttH} is obtained by adding in quadrature the statistical and systematic uncertainties as described above and the error of the ILC branching ratio measurement. The combination of the $b\bar{b}$ and W^+W^- final states is performed by

$$\left(\frac{\Delta g_t}{g_t}\right)^{-2}_{\mathrm{comb.}} = \left(\frac{\Delta g_t}{g_t}\right)^{-2}_{W^+W^-} + \left(\frac{\Delta g_t}{g_t}\right)^{-2}_{b\bar{b}} . \quad (1)$$

The relative accuracies of the top quark Yukawa coupling g_t achievable are summarised in Tables 6 and 7.

Table 6. Expected relative error of the top Yukawa coupling g_t from the rate measurement including all systematic uncertainties (statistical errors only) at the LHC for 30 fb^{-1} and from the branching ratio measurement at the ILC

m_H		30 fb^{-1}	
(GeV/c^2)	bb	WW	bb $\oplus$ WW
100	0.22(0.12)		
110	0.25(0.15)		
120	0.31(0.19)	0.52(0.49)	0.27(0.18)
130	0.43(0.28)	0.28(0.25)	0.23(0.19)
140	0.72(0.50)	0.20(0.17)	0.19(0.16)
150		0.18(0.14)	
160		0.16(0.13)	
170		0.17(0.13)	
180		0.18(0.15)	
190		0.22(0.19)	
200		0.26(0.23)	

Table 7. Expected relative error of the top Yukawa coupling g_t from the rate measurement including all systematic uncertainties (statistical errors only) at the LHC for 300 fb^{-1} and from the branching ratio measurement at the ILC

m_H	300 fb^{-1}		
(GeV/c^2)	bb	WW	bb ⊕ WW
100	0.15(0.07)		
110	0.17(0.08)		
120	0.19(0.10)	0.38(0.31)	0.17(0.10)
130	0.26(0.15)	0.20(0.15)	0.16(0.11)
140	0.41(0.26)	0.15(0.10)	0.14(0.09)
150	1.88(1.21)	0.14(0.08)	0.14(0.08)
160		0.13(0.07)	
170		0.13(0.07)	
180		0.14(0.08)	
190		0.15(0.10)	
200		0.16(0.11)	

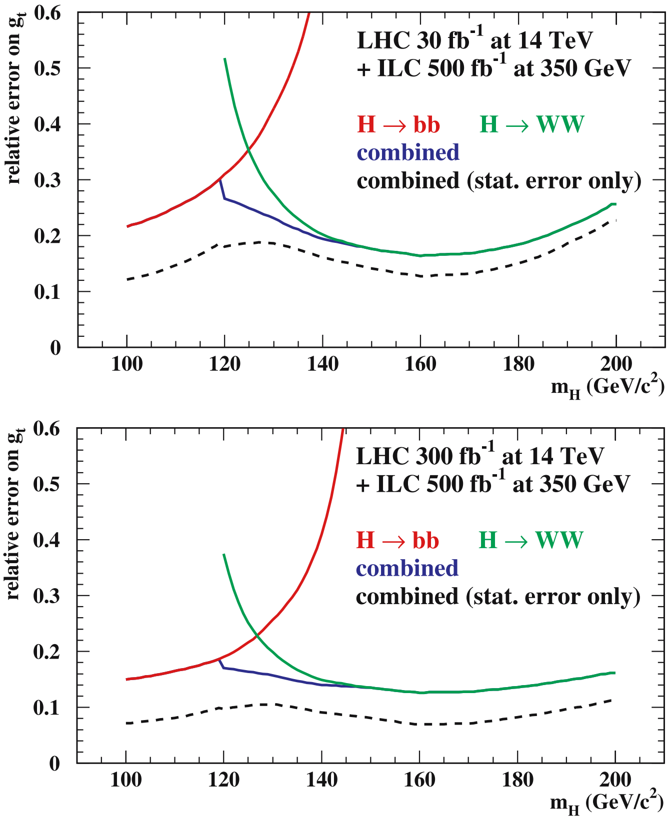

Fig. 1. Achievable precision of the top Yukawa coupling from 30 fb^{-1} at the LHC and 500 fb^{-1} at the ILC (*upper*), and from 300 fb^{-1} at the LHC and 500 fb^{-1} at the ILC (*lower*). The *red curve* shows the precision obtainable from the $H^0 \to b\bar{b}$ final state, the *green curve* that from the $H^0 \to W^+W^-$ final state and the *blue curve* that from the combination of the two. The *dashed lines* show the expected precision taking into account only statistical errors

In Fig. 1 the relative accuracies from the $H^0 \to b\bar{b}$ and $H^0 \to W^+W^-$ channels are shown individually and combined for both of the two integrated luminosities of 30 fb^{-1} and 300 fb^{-1} at the LHC, respectively. Also shown are the results which would be obtained when all systematic errors were neglected.

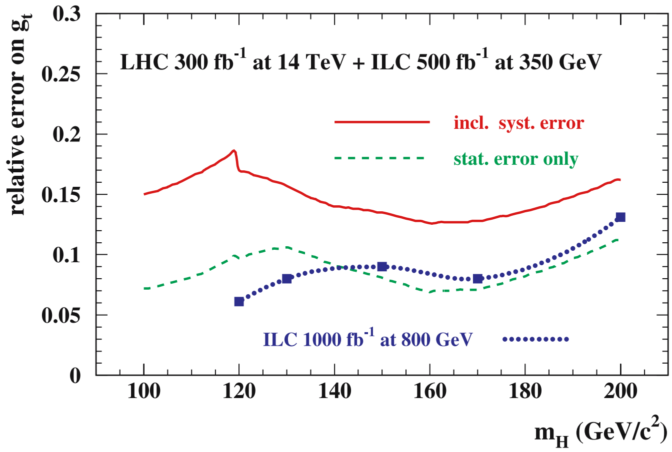

Fig. 2. Achievable precision of the top Yukawa coupling from 300 fb^{-1} at the LHC and 500 fb^{-1} at the ILC at 350 GeV taking into account all systematic uncertainties (*solid red curve*) and using only statistical errors (*dashed green curve*). For comparison, the expected precision from 1000 fb^{-1} at the ILC at 800 GeV alone (*dotted blue curve*) is also shown [18]

For 300 fb^{-1} at the LHC and 500 fb^{-1} at the ILC the obtainable relative uncertainty is approximately 15% for a Higgs boson mass between 120 and 200 GeV/c^2. The purely statistical uncertainty ranges from 7% to 11% as shown in Fig. 2. The size of the obtained uncertainties is comparable to those obtained for the LHC alone [11], which however require model-dependent assumptions.

In Fig. 2 we also show the precision which can be achieved at the ILC alone if operated at 800 GeV centre-of-mass energy [18] from the measurement of the $e^+e^- \to t\bar{t}H^0$ process with $H^0 \to b\bar{b}$ and $H^0 \to W^+W^-$ combined.

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